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Faint Young Sun, Planetary Paleoclimates and Varying Fundamental Constants

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The effect of a cosmic time variation of the gravitational constant on the solar luminosity evolution is studied. It is demonstrated that a varying gravitational constant can substantially affect the solar flux at the planetary orbits on geological time scales. Mean surface temperatures well above the freezing point of water can be achieved in this way throughout the Archean and Hadean, without invoking an increased greenhouse effect or a lower albedo. Instead of a monotonous decline of the solar flux in look-back time, due to a dim early Sun, we infer a flux minimum during the Early Proterozoic and Late Archean. In this epoch, the solar flux is capable of generating mean surface temperatures between 7°C and 12°C, as compared to the present 15°C. The flux then steadily increases, culminating in temperatures between 12°C and 19°C some 4.5 Gry ago, depending on the parameters chosen for the 'standard' Sun. This explains the absence of polar caps, and even warm oceans in the Archean and Hadean are possible at these temperatures. No change of the present 33 K greenhouse effect is required. As for Mars, we show that the solar flux at the Martian orbit before 3.8 Gyr was at least 90% of the present-day flux, so that mean surface temperatures above the freezing point could have been generated by CO_2 greenhouse warming. The time variation of the gravitational constant is such that the moderate dimensionless ratio $\hbar^2 H_0/(k_0 c m_\pi^3)$ stays constant in cosmic time. There are stringent bounds on the logarithmic time derivative of the gravitational constant from lunar laser ranging and helioseismology, which indicate that the first-order derivative at the present epoch is too small to noticeably affect the solar luminosity evolution within the age of the Earth. However, higher-order derivatives have to be taken into account, as they do affect the solar flux in geologic look-back time. We consider the impact of a varying gravitational constant on the redshift scaling of the linear size of radio galaxies. The observed scaling exponent also enters the solar luminosity evolution. The age of the universe has a substantial imprint on planetary paleoclimates.

KEY WORDS: cosmic time; radio galaxies; solar evolution; prebiotic Earth; paleoclimatology.

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1. INTRODUCTION

We point out evidence from planetary paleoclimates for a cosmic time variation of the gravitational constant. The ‘early faint Sun problem’ (Sagan and Chyba, 1997) is reanalyzed, that is, the difficulty to reconcile Earth’s high surface temperatures during the Archean and Hadean and the evidence for fluid water on Mars prior to 3.8 Gyr with a weak solar luminosity. A varying gravitational constant reduces planetary orbital radii and increases the luminosity of the early Sun. The combined effect results in an increase of the solar flux at the planetary orbits (Teller, 1948).

There are bounds from lunar laser ranging and helioseismology on the present logarithmic time derivative $\dot{k}_0/k_0$ of the gravitational constant, which seem at first sight to exclude a substantial impact of a varying k on the solar luminosity evolution (Newman and Rood, 1977). However, we carry out a systematic analysis of this time variation, and show that it can have a pronounced effect on planetary paleoclimates, even if the first derivative of k is negligible at the present epoch. The reasoning is based on a simple fact. If we put the solar evolution into a cosmological context, that is, relate it to cosmic time and the space expansion, we have to cover a period of some 4.6 Gyr, which is currently believed to be about one third of the age of the universe. We show that linearization over a look-back time of that magnitude is not an option, and that the higher-order logarithmic derivatives of k significantly brighten the young Sun.

The time variation of the gravitational constant is chosen proportional to the Hubble parameter, $k(\tau) \propto H(\tau)$, so that the moderate dimensionless ratio $\hbar^2 H/(kcm_\pi^3) \approx 1/(4\pi)$ stays constant in cosmic time. Orbital radii adiabatically scale $\propto 1/H(\tau)$, and so do galaxy diameters. Based on this $k(\tau)$, we calculate the cosmic time evolution of the solar luminosity and flux, using Schwarzschild’s (1958) scaling argument. In the case of constant k, luminosity and flux steadily decrease in look-back time, resulting in negative surface temperatures (Sagan and Mullen, 1972; Sagan, 1977). However, there is no evidence for glaciers or polar caps in the Archean, before 2.5 Gyr (Kasting, 1989), and oceans may already have existed for most part of the Hadean, prior to 3.8 Gyr ago (Wilde *et al.*, 2001). We show that a varying gravitational constant can alter the solar flux quite substantially on geological time scales. The flux at the Earth’s orbit initially decreases in look-back time, reaches a minimum in the Early Proterozoic or Late Archean, 2–2.8 Gyr ago, and then steadily increases throughout the Archean and Hadean. This results in mean surface temperatures at 4.5 Gyr close to or even above the present 15°C, assuming a constant 33 K greenhouse shift. At the Martian orbit, a look-back/present-day flux ratio of $S_1/S_0 > 0.9$ can be maintained during the heavy-bombardment period, before 3.8 Gyr, which is sufficient to generate surface temperatures above the melting point of water by virtue of CO_2 greenhouse warming (Kasting, 1991).

A time variation of the gravitational constant affects planetary orbits and stellar luminosities as well as the redshift scaling of the linear and angular diameters of galaxies (Sandage, 1988; Petrosian, 1998). We work this out for a specific class of expansion factors, representing the crossover between power-law expansion, $\propto \tau^{\alpha+\beta}$, in the early stage and exponential expansion, $\propto \tau^{\beta} e^{\alpha\eta\tau}$, in the late stage of the cosmic evolution. We relate the scaling exponent of the linear size of radio galaxies to the power-law index $\alpha + \beta$. This index in turn affects the early solar flux and the climate evolution. We also demonstrate that the age of the universe has a pronounced impact on the time evolution of the solar flux, which shows in the surface temperatures. We consider cosmic ages three to four times the solar system age, 4.6 Gyr.

In Section 2, we derive the solar luminosity evolution in cosmic time. We explain how the varying gravitational constant $k(\tau)$ connects to the Hubble parameter $H(\tau)$, and relate its present-day logarithmic derivative to the deceleration parameter. We study the effect of a varying k on the redshift scaling of galaxy diameters (linear & angular size) and relate the scaling exponent of radio galaxies to the power-law index of the expansion factor. We discuss observational results in this regard, which roughly point toward $\alpha + \beta \approx 0.5$.

In Section 3, we determine the effect of a varying k on the solar flux at the planet's orbit, in particular the resulting black-body equilibrium temperatures, and calculate the flux minimum in look-back time. In Section 4, we study Earth's mean surface temperature at the flux minimum and at other look-back times in the range 2–4.5 Gyr, and compare with constant k. We discuss the effect of the cosmic age on these surface temperatures, and relate them to the power-law index $\alpha + \beta$ of the asymptotic expansion factor mentioned above. We show that surface temperatures well above the freezing point of water can be reached, throughout the Archean and Hadean period, and enumerate the geological evidence for this. In the tables, we demonstrate that the prediction of a warm but not too hot paleoclimate on Earth and Mars by virtue of a varying gravitational constant is quite stable, both with regard to the solar input parameters and the choice of the cosmic expansion factor. In Section 5, we present our conclusions.

2. SOLAR LUMINOSITY EVOLUTION

The following discussion of the 'faint young Sun paradox' (Gilliland, 1989) is based on a simple scaling relation (Schwarzschild, 1958). The solar luminosity relates to the gravitational constant k and the mean molecular weight μ as

$$L(\tau) \propto (k(\tau)\mu(\tau))^{\lambda}, \tag{2.1}$$

with scaling exponent $\lambda \approx 7.5$ (Newman and Rood, 1977; Christensen-Dalsgaard, 1998). The molecular weight connects to the mean hydrogen mass fraction via $\mu \approx 4/(3 + 5X)$, and the time variation of X relates in turn to the solar luminosity as,

$$\frac{\mathrm{d}X}{\mathrm{d}\tau} = -\frac{L}{\varepsilon_{\mathrm{pp}} M}, \tag{2.2}$$

where τ denotes cosmic time. The present-day luminosity, the solar mass, the energy set free by hydrogen burning, and the hydrogen mass fraction are $L_0 \approx 3.83 \times 10^{33}$ erg/s, $M \approx 1.99 \times 10^{33}$ g, $\varepsilon_{pp} \approx 6.34 \times 10^{18}$ erg/g, and $X_0 = 0.735$, respectively (Wasserburg, 1987; Grevesse *et al.*, 1996; Grevesse and Sauval, 1998). Mild variations of these parameters can be accommodated in the subsequent analysis, certainly all solar standard models qualify, cf. after (2.21). The luminosity evolution $L(\tau)$ is obtained by solving (2.1) and (2.2) with the indicated input parameters. We need to know, however, the time variation of the gravitational constant in (2.1), to be inferred from the space expansion.

To this end, we start with the ascending series of the cosmic expansion factor in the Robertson-Walker line element (Sandage, 1988),

$$\frac{a(\tau)}{a_0} = 1 + \Delta - \frac{q_0}{2}\Delta^2 + \frac{p_0}{6}\Delta^3 + \ldots, \tag{2.3}$$

using the shortcut $\Delta := H_0 \cdot (\tau - \tau_0)$. Subscript zeros refer to the present epoch τ_0, so that $H_0 = H(\tau_0)$, where $H(\tau) = \dot{a}(\tau)/a(\tau)$ is the Hubble parameter. The second order in (2.3) is determined by the (deceleration) parameter $q_0 = -\ddot{a}_0 a_0/\dot{a}_0^2$, and the third by $p_0 = a_0^{(3)} a_0^2/\dot{a}_0^3$. These parameters, q_0 and p_0, are directly measurable, as they enter into angular diameters, surface brightness, source counts, *etc*.

The present-day gravitational constant is $k_0 \approx 6.707 \times 10^{-45} \hbar c^5\,\mathrm{MeV}^{-2}$. For the moderate ratio

$$\frac{\hbar^2 H_0}{k_0\, c m_\pi^3} \approx \frac{1}{4\pi} \tag{2.4}$$

to stay constant in the cosmic evolution, the time variation of the gravitational constant has to be proportional to the Hubble parameter, $k(\tau) \propto H(\tau)$, so that

$$\frac{k(\tau)}{k_0} = \frac{H(\tau)}{H_0}. \tag{2.5}$$

The pion mass m_π is $139.567\,\mathrm{MeV}/c^2$. To satisfy (2.4), we choose $H_0 = h_0/(9.778\,\mathrm{Gyr})$, with $h_0 \approx 0.6802$, or $H_0^{-1} \approx 14.375\,\mathrm{Gyr}$ (Tomaschitz, 1998b). In fact, there seems to be a trend closing in on $h_0 \approx 0.68$ (Melchiorri *et al.*, 2003). It is quite remarkable that the ratio (2.4) can be related to the unit sphere, given the very large numbers involved. However, it is not really important to do so in the subsequent investigations; a variation of h_0 within a few percent can easily be accommodated.

Remarks: The speed of light, the Planck constant, and the pion mass do not vary in cosmic time. There is, however, the possibility to possibility to scale these constants with cosmic time as an alternative to the space expansion (Tomaschitz, 1998b). That is, instead of expanding the intergalactic distances by varying the length unit of the 3-space (as defined by the curvature radius, for instance), we may equally well contract the measuring rods by assuming a time variation of the fundamental constants. Certain moderate dimensionless ratios enumerated in Tomaschitz (2000) have to stay constant, so that nuclear and electromagnetic interactions scale at the same rate. The ratio (2.4) is reminiscent of the electric fine structure constant, which is kept constant to avoid dispersion in redshifts, which would otherwise show in a broadening of spectral lines (Zel'dovich, 1964; Steigman, 1978; Tomaschitz, 1993, 1994, 1998a). Finally, one would expect a time variation of the gravitational constant to manifest on geological time scales, given that the age of the Earth already covers one fourth to one third of the cosmic age as inferred from main-sequence fitting of globular clusters and nuclear chronometers in halo giants, cf. Section 4.

We study a specific class of expansion factors,

$$a(\tau) = A\tau^{\beta}\sinh^{\alpha}(\eta\tau/\tau_0), \tag{2.6}$$

describing the cross-over from an initial power-law, $\propto \tau^{\alpha+\beta}$, to exponential expansion, $\propto \tau^{\beta}e^{\alpha\eta\tau}$, in the final stage. The normalization A can be chosen arbitrarily, we may fix it by the convention $a_0 := a(\tau_0) = 1$ at the present epoch. The constants α and η are positive, and $\alpha + \beta \geq 0$. The latter condition is required for expansion, $\dot{a}(\tau) > 0$, throughout the cosmic evolution, $0 < \tau < \infty$. The logarithmic derivative of (2.6) reads,

$$H(\tau) = \frac{\dot{a}(\tau)}{a(\tau)} = \frac{\alpha\eta}{\tau_0}\coth\left(\eta\frac{\tau}{\tau_0}\right) + \frac{\beta}{\tau}. \tag{2.7}$$

In the asymptotic regimes $\tau \to 0, \infty$, this is evidently positive if $\alpha + \beta \geq 0$, and it stays so for finite τ. In fact, if $H(\tau)$ were negative for a finite τ, there would be a minimum defined by $\dot{H}(\tau_{\min}) = 0$, so that $H(\tau_{\min}) < 0$, and $\beta < 0$. However, by virtue of $\dot{H}(\tau_{\min}) = 0$, we can equate $\coth(\eta\tau_{\min}/\tau_0)$ in (2.7) to an algebraic function, and easily check that $H(\tau_{\min}) > 0$ for all extrema $\dot{H}(\tau_{\min}) = 0$, cf. (3.4). We require $\alpha + \beta > 0$, so that $H(\tau \to 0) \sim (\alpha + \beta)/\tau$ and $H(\tau \to \infty) \sim \alpha\eta/\tau_0$.

The parameters τ_0, q_0 and p_0 in the ascending series (2.3) relate to the expansion factor (2.6) as

$$H_0\tau_0 = \alpha\eta\coth\eta + \beta, \tag{2.8}$$

$$\varepsilon := q_0 + 1 = 1 - \frac{\ddot{a}_0}{a_0}\frac{a_0^2}{\dot{a}_0^2} = \frac{1}{(H_0\tau_0)^2}\left(\frac{\alpha\eta^2}{\sinh^2\eta} + \beta\right), \tag{2.9}$$

$$\delta := p_0 - 1 = \frac{a_0^{(3)}}{a_0}\frac{a_0^3}{\dot{a}_0^3} - 1 = \frac{2}{(H_0\tau_0)^3}\left(\alpha\eta^3\frac{\coth\eta}{\sinh^2\eta} + \beta\right) - 3\varepsilon. \tag{2.10}$$

The first and second order of the ascending series of the Hubble parameter are determined by ε, δ and τ_0,

$$\frac{H(\tau)}{H_0} = 1 - \varepsilon\Delta + \frac{1}{2}(\delta + 3\varepsilon)\Delta^2 + \cdots, \tag{2.11}$$

where $\Delta = H_0 \cdot (\tau - \tau_0)$ as in (2.3).

Remarks: We have defined the Hubble constant as well as the deceleration parameter completely detached from the Einstein equations. The natural way to deal with varying fundamental constants is to adopt an absolute cosmic space-time conception, as they break the covariance of the Lagrangians. Once general covariance is abandoned, evolution equations based on the Riemann tensor or its generalizations lose their motivation (Tomaschitz, 1998c). A cosmic time variation of the fundamental constants is remote from relativity principles, as it establishes a relation of local Lagrangians to the absolute cosmic time of the comoving galaxy frame (Tomaschitz, 2004). Nevertheless, on local time scales and restricted to locally geodesic neighborhoods, the cosmic time dependence of the fundamental constants can be neglected, so that Lorentz invariance is preserved.

If ε, δ and τ_0 are taken as input, the parameters α, β and η in the expansion factor (2.6) can be recovered by inversion of (2.8)–(2.10). First, η is found by solving

$$\frac{4\eta(\eta \coth \eta - 1)}{\sinh(2\eta) - 2\eta} = \frac{(H_0\tau_0)^2(\delta + 3\varepsilon) - 2H_0\tau_0\varepsilon}{1 - H_0\tau_0\varepsilon}, \tag{2.12}$$

Once η is determined by (2.12), we find the remaining parameters in the expansion factor (2.6) as

$$\frac{\alpha}{H_0\tau_0} = 2\frac{\sinh^2 \eta}{\eta}\frac{1 - H_0\tau_0\varepsilon}{\sinh(2\eta) - 2\eta}, \tag{2.13}$$

$$\frac{\beta}{H_0\tau_0} = -\frac{2\eta - H_0\tau_0\varepsilon \sinh(2\eta)}{\sinh(2\eta) - 2\eta}. \tag{2.14}$$

The mentioned condition, $\alpha + \beta \geq 0$, to ensure expansion throughout the evolution, cf. after (2.7), is thus equivalent to

$$H_0\tau_0\varepsilon \geq \frac{\eta^2 - \sinh^2 \eta}{(\eta \coth \eta - 1)\sinh^2 \eta}. \tag{2.15}$$

As $\sinh \eta > \eta$ and $\eta \coth \eta > 1$ hold true for positive η, the right-hand side is negative and thus $\varepsilon \geq 0$ is always permissible.

If the parameters τ_0, $q_0 = -1 + \varepsilon$, and $p_0 = 1 + \delta$, which define the Taylor coefficients of the expansion factor (2.3), are the observed input, the parameters α, β and η in (2.6) can be recovered via (2.12)–(2.14). If we take $\alpha + \beta =: \gamma$ as input instead of p_0, together with ε and τ_0, we may replace relations (2.8) and

(2.9) by

$$\frac{\gamma}{2H_0\tau_0}\frac{\eta(\sinh(2\eta)-2\eta)}{\sinh^2\eta-\eta^2}+H_0\tau_0\varepsilon\frac{\sinh^2\eta(1-\eta\coth\eta)}{\sinh^2\eta-\eta^2}=1, \tag{2.16}$$

$$\frac{\alpha}{H_0\tau_0}=\frac{1-\gamma/(H_0\tau_0)}{\eta\coth\eta-1},\quad \frac{\beta}{H_0\tau_0}=\frac{1}{\eta\coth\eta-1}\left(\frac{\gamma}{H_0\tau_0}\eta\coth\eta-1\right), \tag{2.17}$$

and solve (2.16) for η. p_0 is then calculated via (2.10) or (2.12). H_0 and τ_0 only enter as dimensionless product in the above identities. A first hint on the value of γ is obtained from the linear and angular diameters of radio galaxies, from their redshift scaling, that is.

Angular diameters are defined as the ratio $\theta = y(\tau_1)/d(\tau_1)$, where $y(\tau_1)$ the intrinsic galaxy diameter and $d(\tau_1)$ the metric look-back distance. Both the linear size of the source and the look-back distance are taken at emission time τ_1. The time evolution of galaxy diameters is the same as of planetary orbital radii, inversely proportional to the gravitational constant, so that $y(\tau_1) = y_0H_0/H(\tau_1)$, where y_0 is the present-day diameter. This follows from the virial theorem and the adiabatic time scaling of the Newtonian potential (Teller, 1948). This scaling applies to galaxies, it may not be valid for other extended radio sources, let alone compact sources. The look-back distance can readily be calculated from the Robertson-Walker line element, $d(\tau_1) = ca(\tau_1)\int_{\tau_1}^{\tau_0} a^{-1}(\tau)d\tau$, where present epoch τ_0 and emission time τ_1 connect via $1+z = a(\tau_0)/a(\tau_1)$. When considering high z in leading-order asymptotics, it suffices to approximate the expansion factor (2.6) by $a(\tau) \propto \tau^\gamma$, $\gamma := \alpha+\beta$, so that $\tau_1 \propto z^{-1/\gamma}$. The linear size scales as $y \propto z^{-1/\gamma}$, since $H(\tau) \propto 1/\tau$, cf. after (2.7); we use the more customary notation $n = 1/\gamma$ for this scaling exponent. If $0 < \gamma < 1$, the above integral defining the look-back distance converges, and is thus in leading order independent of z. In this way, we find the high-redshift scaling of the angular diameter as $\theta \propto z^{1-n}$. If $\gamma > 1$, we find $\theta \propto 1$, independent of γ in leading order, and if $\gamma = 1$, the logarithmic divergence of the integral in $d(\tau_1)$ shows as $\theta \propto 1/\log z$. The low-z scaling can easily be extracted from the expansion (2.3), applicable to short look-back intervals $\tau_0 - \tau_1$, and we find in leading order, $\theta \propto z^{-1}$.

We shortly list the observed scaling exponents of the linear size, customarily defined by $y \propto (1+z)^{-n}$ (Petrosian, 1998; Maloney and Petrosian, 1999; Lubin and Sandage, 2001). An exponent of $n \approx 3 \pm 0.5$ is quoted in Kapahi (1989) and Singal (1993), assuming a luminosity scaling of $y \propto L^{0.3}$ at constant z, and similarly in Oort *et al.* (1987), $n \approx 3.3 \pm 0.5$, with the same luminosity dependence. The luminosity evolution is empirical, the exponent serves as a further fitting parameter. On the other hand, no linear-size evolution with redshift was found in Nilsson *et al.* (1993). In between these extreme cases, an exponent of

$n \approx 1.5 \pm 1.4$ is cited in Barthel and Miley (1988) for a quasar sample extending to $z \leq 2.7$, where only a very weak luminosity dependence, $y \propto L^{-0.03}$, is needed. An exponent of $n \approx 1.7 \pm 0.4$ (with $y \propto L^{-0.06}$ and $z \leq 1$) is inferred in Neeser *et al.* (1995), where also a sample of radio galaxies with quasars discarded is studied. In this reduced sample, $n \approx 1.96^{+0.43}_{-0.49}$. This is a crucial point, as the intrinsic size evolution inversely proportional to the Hubble parameter is justified for galaxies only, hinging on the virial theorem. No substantial linear size evolution of quasars was found in Singal (1993), which suggests that they are based on a different interaction mechanism. Though these exponents are still somewhat vague, they point toward a γ-range centered at $\gamma \approx 0.5$ or $n \approx 2$.

We continue the discussion of (2.5), the proportionality $k(\tau) \propto H(\tau)$. An excellent estimate of the deceleration parameter q_0 is obtained from bounds on the present-day logarithmic derivative of k, such as $|\dot{k}_0/k_0| < 8 \times 10^{-3}\,\mathrm{Gyr}^{-1}$, inferred from lunar laser ranging (Williams *et al.*, 1996). The best bound so far comes from helioseismology, $1.6 \times 10^{-3}\,\mathrm{Gyr}^{-1}$ (Guenther *et al.*, 1998). This suggests that q_0 is very close to -1, by virtue of, cf. (2.5) and (2.11),

$$\dot{k}_0/k_0 = -H_0(1 + q_0). \tag{2.18}$$

The seismological bound gives $|\varepsilon| < 0.023$, where $\varepsilon = 1 + q_0$, cf. (2.9). We study an expansion factor where ε is one order below this bound, cf. (3.3), and for the most part of this paper we even put $\varepsilon \approx 0$, cf. Tables I–VI. The goal is to demonstrate that the time variation of k has a significant impact on the solar luminosity evolution and is quite capable of enhancing the luminosity of the early Sun, even if $\dot{k}_0/k_0 \approx 0$ at the present epoch. We do not even need to be particularly selective with regard to input parameters, solar and others, to argue a bright young Sun and a warm paleoclimate. If $\dot{k}_0/k_0$ is very small as suggested by the above

Table I. Parameters relating to the cosmic expansion factor, $a(\tau) \propto \tau^{\beta} \sinh^{\alpha}(\eta\tau/\tau_0)$, cf. (2.6). $q_0 = -1 + \varepsilon$ and $p_0 = 1 + \delta$ define the second and third order of the ascending series of $a(\tau)$, cf. (2.3). η and $\varepsilon = 0$ are input parameters, whereas δ, α and β are calculated via (2.12)–(2.14). The latter have been rescaled with $H_0\tau_0$ to make them independent of the present epoch τ_0. The coefficients $c_{3,4}$ are calculated via (2.23) and the parameters in this table. c_3 is independent of λ since $\varepsilon = 0$. At $\lambda = 7.5$ (scaling exponent of the luminosity in (2.1)), the c_i determine the third and fourth order of the ascending series of K_1/Δ_1 in look-back time, cf. (2.24) and Table II. At $\lambda = 1$, they determine the solar flux minimum, cf. (3.8)

η	$\alpha/(H_0\tau_0)$	$-\beta/(H_0\tau_0)$	$(H_0\tau_0)^2\delta$	c_3	$c_4^{(\lambda=1)}$	$c_4^{(\lambda=7.5)}$
0.5	6.19951	5.70772	0.93580	0.49851	0.49265	1.20417
1	1.69787	1.22936	0.76967	0.48034	0.47748	0.95879
1.5	0.86139	0.42748	0.56187	0.42588	0.45539	0.71190
2	0.56479	0.17175	0.36913	0.33847	0.41169	0.52240
2.5	0.42316	0.07225	0.22165	0.24113	0.34042	0.38034
3	0.34185	0.03066	0.12354	0.15603	0.25491	0.26731
3.5	0.28888	0.01293	0.06482	0.09325	0.17425	0.17766

Table II. Input from Table I. In addition, $H_0\tau_0 = 1.1$, implying a present epoch $\tau_0 \approx 15.8$ Gyr Tables II–VI are compiled for a look-back time $\tau_0 - \tau_1$ of 3.8 Gyr, so that $\Delta_1 = H_0 \cdot (\tau_1 - \tau_0) \approx -0.264$. k_0 is the present-day gravitational constant, k_1 its look-back value at τ_1, and K_1 determines the look-back/present-day luminosity ratio, cf. (3.1) and Table III. The ratio k_1/k_0 is calculated via (3.2), and K_1/Δ_1 via (2.24), with $\lambda = 7.5$ and the $c_{3,4}^{(\lambda=7.5)}$ in Table I. The present-day/look-back ratio R_0/R_1 of a planetary orbital radius is identical with k_1/k_0, cf. after (3.5)

η	k_1/k_0 ($H_0\tau_0 = 1.1$)	K_1/Δ_1 ($H_0\tau_0 = 1.1$)
0.5	1.03599	1.08629
1	1.03089	1.07264
1.5	1.02404	1.05504
2	1.01710	1.03795
2.5	1.01124	1.02410
3	1.00689	1.01427
3.5	1.00401	1.00797

bounds, the first order of the Taylor expansion (2.11) of the Hubble parameter is negligible, but the second order, $\sim\delta\Delta^2/2$, has to be taken into account for a look-back interval $\tau_0 - \tau_1$ comparable to the age of the solar system, this is the all important point.

Returning to the luminosity evolution defined by (2.1) and (2.2), we find

$$\frac{1}{\mu}\frac{d\mu}{d\tau} \approx \frac{5}{(3 + 5X_0)}\frac{L}{\varepsilon_{\mathrm{pp}}M}, \tag{2.19}$$

$$\frac{\dot{L}}{L} = \xi\frac{L}{\varepsilon_{\mathrm{pp}}M} + \lambda\frac{\dot{k}}{k}, \quad \xi \approx \frac{12.5}{1 + (5/3)X_0} \approx 5.62, \tag{2.20}$$

Table III. Input as in Tables I and II, in particular $H_0\tau_0 = 1.1$ and $\tau_0 - \tau_1 = 3.8$ Gyr. The exponent $n = 1/(\alpha + \beta)$ determines the high-redshift scaling of angular diameters, $\theta \propto z^{1-n}$, subject to the varying gravitational constant, cf. after (2.17). The other entries relate to the solar luminosity. L_1/L_0 is the look-back/present-day luminosity ratio, S_1/S_0 the corresponding flux ratio (solar flux at the planet's orbit), and T_1/T_0 (in K) is the resulting equilibrium temperature ratio (black-body, the same for all planets, without inclusion of greenhouse effect and albedo), cf. the discussion following (3.3). These ratios are calculated via (3.1) (with $\lambda = 7.5$) and Table II, for two choices of solar parameter, $\sigma_a \approx 0.774$ and $\sigma_b \approx 1.22$, cf. after (2.21)

η	α	$-\beta$	$n-1$	$L_1/L_0(\sigma_a)$	$S_1/S_0(\sigma_a)$	$T_1/T_0(\sigma_a)$	$L_1/L_0(\sigma_b)$	$S_1/S_0(\sigma_b)$	$T_1/T_0(\sigma_b)$
0.5	6.819	6.278	0.848	1.06687	1.1450	1.0344	0.96578	1.0366	1.0090
1	1.868	1.352	0.941	1.03042	1.0950	1.0230	0.93369	0.9923	0.9981
1.5	0.947	0.470	1.095	0.98311	1.0310	1.0076	0.89196	0.9354	0.9834
2	0.621	0.189	1.313	0.93689	0.9692	0.9922	0.85108	0.8804	0.9687
2.5	0.465	0.079	1.591	0.89926	0.9196	0.9793	0.81772	0.8362	0.9563
3	0.376	0.034	1.921	0.87207	0.8841	0.9697	0.79357	0.8045	0.9471
3.5	0.318	0.014	2.294	0.85446	0.8613	0.9634	0.77791	0.7842	0.9410

Table IV. Input as in Table I; the caption of Table II applies. Here and in Tables V and VI, we study the effect of the parameter $H_0\tau_0$ on the time variation of k and the luminosity, flux and temperature ratios, as well as on the scaling exponent of the angular diameter. In Tables II and III, we considered $H_0\tau_0 = 1.1$. Here, we study two further values, $H_0\tau_0 = 0.95$, that is, a cosmic age of $\tau_0 \approx 13.7$ Gyr, as well as $H_0\tau_0 = 1.3$, so that $\tau_0 \approx 18.7$ Gyr. τ_0 enters the listed quantities only through the dimensionless product $H_0\tau_0$. The look-back time is in both cases 3.8 Gyr or $\Delta_1 \approx -0.264$

η	k_1/k_0 ($H_0\tau_0 = 0.95$)	K_1/Δ_1 ($H_0\tau_0 = 0.95$)	k_1/k_0 ($H_0\tau_0 = 1.3$)	K_1/Δ_1 ($H_0\tau_0 = 1.3$)
0.5	1.05090	1.12117	1.02452	1.05914
1	1.04396	1.10220	1.02091	1.04967
1.5	1.03456	1.07774	1.01612	1.03747
2	1.02491	1.05393	1.01132	1.02568
2.5	1.01664	1.03450	1.00733	1.01618
3	1.01039	1.02060	1.00442	1.00950
3.5	1.00615	1.01160	1.00252	1.00526

where $\lambda \approx 7.5$. A simple rescaling, $\tilde{L} = Lk^{-\lambda}$, gives $d\tilde{L}/\tilde{L}^2 = \xi k^{\lambda}(\tau)d\tau/(\varepsilon_{\mathrm{pp}}M)$, solved by

$$\frac{L(\tau)}{L_0} = \frac{(k(\tau)/k_0)^{\lambda}}{1-\sigma K(\tau)}, \quad K(\tau) := \frac{H_0}{k_0^{\lambda}} \int_{\tau_0}^{\tau} k^{\lambda}(\tau)d\tau, \quad \sigma := \frac{L_0\xi}{\varepsilon_{\mathrm{pp}}MH_0}. \tag{2.21}$$

We use the shortcuts $K_1 = K(\tau_1)$ and $L_{0,1} = L(\tau_{0,1})$, where τ_0 stands for the present epoch, and $\tau_0 - \tau_1$ is the look-back time. The gravitational constant $k(\tau)$ is determined by (2.5) and (2.7). The solar parameters stated after (2.2) give $\sigma \approx 0.774$. This dimensionless constant is somewhat uncertain depending on the solar modeling (Sackmann and Boothroyd, 2003). A popular choice in paleoclimatic studies is $\sigma \approx 1.22$, resulting in an even dimmer early Sun (Gough, 1981; Gilliland, 1989). In the tables, we discuss both values of σ, denoted by $\sigma_{\mathrm{a,b}}$, respectively.

The integral $K(\tau_1)$ over the varying gravitational constant in (2.21) can be calculated from the ascending series of $k^{\lambda}(\tau)$, using term-by-term integration; the convergence is quite rapid for look-back times below the solar age. We use the

Table V. Input taken from Tables I and IV, otherwise the caption of Table III applies, but now with $H_0\tau_0 = 0.95$. The look-back interval is 3.8 Gyr. Luminosity, flux, and temperature ratios are listed twice, for the solar parameters $\sigma_{\mathrm{a,b}}$ stated in Table III

η	α	$-\beta$	$n-1$	$L_1/L_0(\sigma_a)$	$S_1/S_0(\sigma_a)$	$T_1/T_0(\sigma_a)$	$L_1/L_0(\sigma_b)$	$S_1/S_0(\sigma_b)$	$T_1/T_0(\sigma_b)$
0.5	5.890	5.422	1.140	1.18064	1.3039	1.0686	1.06614	1.1774	1.0417
1	1.613	1.168	1.247	1.12694	1.2282	1.0527	1.01900	1.1106	1.0266
1.5	0.818	0.406	1.426	1.05740	1.1318	1.0314	0.95779	1.0251	1.0062
2	0.537	0.163	1.678	0.98957	1.0395	1.0097	0.89789	0.9432	0.9855
2.5	0.402	0.069	2.000	0.93425	0.9656	0.9913	0.84889	0.8774	0.9678
3	0.325	0.029	2.383	0.89414	0.9128	0.9775	0.81328	0.8303	0.9546
3.5	0.274	0.012	2.815	0.86773	0.8784	0.9681	0.78977	0.7995	0.9456

Table VI. As Table V, with $H_0\tau_0 = 1.3$. The enumerated quantities are defined in the caption of Table III

η	α	$-\beta$	$n-1$	$L_1/L_0(\sigma_a)$	$S_1/S_0(\sigma_a)$	$T_1/T_0(\sigma_a)$	$L_1/L_0(\sigma_b)$	$S_1/S_0(\sigma_b)$	$T_1/T_0(\sigma_b)$
0.5	8.059	7.420	0.564	0.98588	1.0348	1.0086	0.89421	0.9386	0.9843
1	2.207	1.598	0.642	0.96166	1.0023	1.0006	0.87284	0.9097	0.9766
1.5	1.120	0.556	0.773	0.93022	0.9604	0.9900	0.84505	0.8725	0.9665
2	0.734	0.223	0.957	0.89953	0.9200	0.9794	0.81788	0.8365	0.9563
2.5	0.550	0.094	1.192	0.87471	0.8876	0.9706	0.79586	0.8076	0.9480
3	0.444	0.040	1.472	0.85688	0.8645	0.9642	0.78002	0.7869	0.9419
3.5	0.376	0.017	1.788	0.84542	0.8497	0.9601	0.76983	0.7737	0.9379

rescaled dimensionless look-back interval $\Delta_1 := H_0 \cdot (\tau_1 - \tau_0)$, cf. after (2.3), as well as the shortcuts $k_{0,1} = k(\tau_{0,1})$. Δ_1 is defined negative; a look-back time of $\tau_0 - \tau_1 \approx 3.8$ Gyr gives $\Delta_1 \approx -0.264$.

We expand $k^\lambda(\tau)$ up to the fourth order in $\Delta = H_0 \cdot (\tau - \tau_0)$, cf. (2.5) and (2.7),

$$\frac{k^\lambda(\tau)}{k_0^\lambda} = 1 - \lambda\varepsilon\Delta + \lambda c_2\Delta^2 - \frac{\lambda c_3\Delta^3}{(H_0\tau_0)^3} + \frac{\lambda c_4\Delta^4}{(H_0\tau_0)^4} + \cdots, \tag{2.22}$$

$$\begin{aligned} c_2 &:= \tfrac{1}{2}(\delta + 3\varepsilon + (\lambda-1)\varepsilon^2), \\ c_3 &:= \frac{\alpha}{H_0\tau_0}\frac{\eta^4}{\sinh^2\eta}\left(\frac{2}{3} + \frac{1}{\sinh^2\eta}\right) + \frac{\beta}{H_0\tau_0} + \frac{1}{2}(\lambda-1)(H_0\tau_0)^3\varepsilon(\delta + \mathrm{O}(\varepsilon)), \\ c_4 &:= \frac{\alpha}{H_0\tau_0}\frac{\eta^5\coth\eta}{\sinh^2\eta}\left(\frac{1}{3} + \frac{1}{\sinh^2\eta}\right) + \frac{\beta}{H_0\tau_0} + \frac{1}{8}(\lambda-1)(H_0\tau_0)^4(\delta^2 + \mathrm{O}(\varepsilon)). \end{aligned} \tag{2.23}$$

This is used in the range $|\Delta| \le |\Delta_1| \le 0.327$, where the numerical bound stems from the solar main-sequence turnoff age, 4.7 Gyr. The third and fourth order coefficients, $c_{3,4}$, have been expanded in ε, for the sake of simplicity, so that the terms stated in (2.22) amount to a double series expansion in fourth order. In the first and second order in Δ, however, there is no ε-expansion involved, and if $\lambda = 1$, there is no ε-expansion in $c_{3,4}$ either. We occasionally write $c_i^{(\lambda)}$, cf. (3.8) and Tables I and VIII. $K(\tau)$ in (2.21) is calculated via term-by-term integration of the expansion (2.22),

$$\frac{K_1}{\Delta_1} = 1 - \frac{1}{2}\lambda\varepsilon\Delta_1 + \frac{1}{3}\lambda c_2\Delta_1^2 - \frac{1}{4}\frac{\lambda c_3\Delta_1^3}{(H_0\tau_0)^3} + \frac{1}{5}\frac{\lambda c_4\Delta_1^4}{(H_0\tau_0)^4} + \cdots \tag{2.24}$$

When compiling the tables in Sections 3 and 4, we use this expansion of $K(\tau_1)$ in the luminosity ratio $L(\tau_1)/L_0$.

3. THE SOLAR FLUX MINIMUM IN LOOK-BACK TIME

We turn to the solar flux at a planetary orbit. The equilibrium condition for black-body radiation is $S \propto T^4$, the flux scaling with the fourth power of temperature. The cosmic time scaling of the flux follows from $S \propto L/R^2$. The proportionality factors are constant, since atomic or nuclear periods (time units) do not scale with time, nor does the Stefan-Boltzmann constant in the T^4-law. The orbital radius scales with the inverse gravitational constant, $R \propto k^{-1}(\tau)$ (Teller, 1948). In this way, we infer the adiabatic time evolution of the equilibrium temperature, $T \propto L^{1/4}k^{1/2}$, or, cf. (2.21),

$$\frac{T_1^4}{T_0^4} = \frac{S_1}{S_0} = \frac{L_1}{L_0}\frac{k_1^2}{k_0^2}, \qquad \frac{L_1}{L_0} \approx \frac{(k_1/k_0)^{\lambda}}{1-\sigma K_1}. \tag{3.1}$$

Here, we use the usual shortcuts $T_{0,1} = T(\tau_{0,1})$, the solar parameter σ is defined in (2.21), and K_1 stands for the expansion (2.24) with $\Delta_1 = H_0 \cdot (\tau_1 - \tau_0)$ and $\lambda \approx 7.5$. In the nominator $(k_1/k_0)^{\lambda}$ of the luminosity ratio, we substitute, cf. (2.5) and (2.7),

$$\frac{k_1}{k_0} = \frac{\alpha}{H_0\tau_0}\eta \coth\left(\left(1+\frac{\Delta_1}{H_0\tau_0}\right)\eta\right) + \frac{\beta}{H_0\tau_0}\frac{1}{1+\Delta_1/(H_0\tau_0)}, \tag{3.2}$$

where $H_0\tau_0$ relates to the expansion factor as stated in (2.8).

The purpose of Tables I–VI is to give a quantitative overview as to how the parameters defining the expansion factor, $a(\tau) \propto \tau^{\beta} \sinh^{\alpha}(\eta\tau/\tau_0)$, cf. (2.6), affect the luminosity, flux and temperature evolution. H_0 is regarded as observed input, specified after (2.5), so that the present epoch τ_0 follows from α, β and η, by virtue of (2.8). If these three parameters are prescribed, the Taylor coefficients $q_0 = \varepsilon - 1$ and $p_0 = \delta + 1$ of the ascending series (2.3) of the expansion factor can be found via (2.9) and (2.10), cf. Table VIII.

In Table I, however, we proceed differently. We start by prescribing an arbitrary value for η, and put $\varepsilon = 0$. The five other entries in this table are calculated from (2.12)–(2.14) and (2.23). (We do not need to calculate the zero of (2.12) as we take η as input.) In Tables II–VI, we specify the third input parameter, $H_0\tau_0$ (apart from η and $\varepsilon = 0$), as well as the look-back time, 3.8 Gyr, or $\Delta_1 = -0.264$, cf. before (2.22). In Tables II and III, we use $H_0\tau_0 = 1.1$, and in Tables IV–VI we calculate the same entries at $H_0\tau_0 = 0.95$ and $H_0\tau_0 = 1.3$. Other values of $H_0\tau_0$ moderately outside this range, and any other look-back interval, $|\Delta_1| < 0.327$, cf. after (2.23), are also admissible, of course. In Tables II and IV, we list k_1/k_0 and K_1, which determine the luminosity, flux and temperature ratios according to (3.1); k_1/k_0 is calculated via (3.2), and K_1 via (2.24). The series expansion (2.24) of K_1 is quite efficient for look-back times up to the solar age. The indicated third and fourth order terms, determined by the coefficients $c_{3,4}$ in Tables I and

VIII, barely affect the luminosity ratio (3.1), unless the cosmic age is very low, cf. Section 4.

The redshift scaling of the linear sizes of radio galaxies discussed before (2.18) suggests to try $\alpha + \beta \approx 0.5$, so that angular diameters scale as $\theta \propto z^{-1}$, at high as well as low redshifts. This in mind, we inspect Table I to find an expansion factor (2.6) determined by the parameters,

$$\alpha = 1, \quad \beta = -\frac{1}{2}, \quad \eta = \frac{3}{2}, \quad a(\tau) \propto \tau^{-1/2} \sinh \frac{3\tau}{2\tau_0}. \tag{3.3}$$

Tables VIII–X are compiled with this expansion factor. We find $H_0\tau_0 \approx 1.1572$, cf. (2.8), resulting in a cosmic age of $\tau_0 \approx 16.54$ Gyr, as well as $\varepsilon \approx -2.786 \times 10^{-3}$, cf. (2.9). The latter determines the present-day logarithmic derivative, $\dot{k}_0/k_0 \approx 1.938 \times 10^{-4}\,\mathrm{Gyr}^{-1}$, according to (2.18). These numbers are based on (3.3). $\dot{k}_0/k_0$ is positive, which indicates that $\dot{k}$ underwent a sign change within the solar system age, as k must have been larger in the past for the planetary orbital radii to be smaller. We want to determine the time τ_k at which this sign change occurred, the zero of $\dot{k}$, that is. According to (2.5), τ_k solves $\dot{H}(\tau_k) = 0$, where the time derivative of the Hubble parameter (2.7) reads,

$$\dot{H}(\tau) = -\frac{\alpha\eta^2}{\tau_0^2} \frac{1}{\sinh^2(\eta\tau/\tau_0)} - \frac{\beta}{\tau^2}. \tag{3.4}$$

This amounts to solve,

$$\frac{\tau_k}{\tau_0} = \frac{\sqrt{-\beta/\alpha}}{\eta} \sinh\left(\eta \frac{\tau_k}{\tau_0}\right), \tag{3.5}$$

where $\beta < 0$, $\alpha + \beta > 0$, and $\eta > 0$, cf. after (2.7). Since $\dot{H}(\tau \to 0) \sim -(\alpha + \beta)/\tau^2$, and $\dot{H}(\tau \to \infty) \sim -\beta/\tau^2$, there is always a unique solution τ_k. If we specify the parameters as in (3.3), we find $\tau_k/\tau_0 \approx 0.9943$, so that the sign change of $\dot{k}_0/k_0$ happened 0.095 Gyr ago.

In Table IX, we list the ratio k_1/k_0, cf. (3.2), for various look-back times, see also Tables II and IV. As the orbits scale inversely proportional to k, we can identify k_1/k_0 with the radial present-day/look-back ratio R_0/R_1. This scaling applies to elliptical orbits, without a change in eccentricity. The look-back radius R_1 at 4.5 Gyr is 3% smaller than the present one. However, since we are past the zero of $\dot{k}$, the orbits have been contracting for the last 95 million years.

The age of the Earth is about one third to one fourth of the cosmic age. A time variation of the gravitational constant should be manifested on geological time scales, otherwise it is not attractive to consider this. A variation of k in the past on the scale of the present-day logarithmic derivative would allow to linearize (2.22), and is therefore too small to effect a change in the paleoclimate (Newman and Rood, 1977). The expansion factor (3.3) gives a $\dot{k}/k$ with a zero close to the present epoch. This is the reason why the value of $\dot{k}_0/k_0$ stated after (3.3) is

Table VII. Solar luminosity for constant k (gravitational constant independent of cosmic time), at different look-back times $\tau_0 - \tau_1$, cf. after (3.3), and normalized with the present-day value L_0. (Tables II–VI all refer to a look-back time of 3.8 Gyr.) Δ_1 is the rescaled look-back interval, $H_0 \cdot (\tau_1 - \tau_0)$. Flux and luminosity ratios coincide, $S_1/S_0 = L_1/L_0$, as the planetary orbital radii stay constant. T_1/T_0 is the ratio of the look-back/present-day equilibrium temperatures, applicable to any planet, as in Tables III, V and VI. T_1^{S} is the look-back surface temperature of Earth, as compared to its present-day mean value, $T_0^{\mathrm{S}} \approx 15°\mathrm{C}$. The entries are calculated as explained after (3.5), for the solar parameters $\sigma_{\mathrm{a,b}}$ stated after (2.21) and in Table III. The first two look-back times are listed for comparison with Table X

$\tau_0 - \tau_1$ (Gyr)	$-\Delta_1$	$L_1/L_0(\sigma_a)$	$T_1/T_0(\sigma_a)$	$T_1^{\mathrm{S}}(\sigma_a)$ (°C)	$L_1/L_0(\sigma_b)$	$T_1/T_0(\sigma_b)$	$T_1^{\mathrm{S}}(\sigma_b)$ (°C)
2.06	0.143	0.90035	0.9741	8.4	0.85145	0.9606	4.9
2.75	0.191	0.87121	0.9661	6.4	0.81102	0.9490	2.0
3.8	0.264	0.83033	0.9546	3.4	0.75638	0.9326	−2.2
4.5	0.313	0.80517	0.9473	1.6	0.72366	0.9223	−4.8

sufficiently small to be consistent with the tight bounds mentioned before (2.18). Yet $\dot{k}/k$ quickly exceeds the present $\dot{k}_0/k_0$ in geological look-back time, so that the nonlinear terms in (2.22) and (2.24) have to be included in the luminosity ratio (3.1). For instance, at a look-back time of 3.8 Gyr, we find $\dot{k}_1/k_1 \approx -0.013\,\mathrm{Gyr}^{-1}$. Beyond the cross-over regime, in the early stage of the cosmic expansion, the logarithmic derivative even diverges, $\dot{k}/k(\tau \to 0) \sim -1/\tau$, cf. (2.5), (2.7) and (3.4); the high-z asymptotics of galaxy diameters hinges upon that.

In Tables VII, IX and X, we consider four look-back times, ranging from 2.06 to 4.5 Gyr. It is instructive to compare the luminosity and temperature ratios listed in Table X with those for constant k in Table VII. The latter implies $k_1 = k_0$ and $K_1 = \Delta_1$ in the luminosity ratio (3.1), so that $S_1/S_0 = L_1/L_0 \approx 1/(1 - \sigma\Delta_1)$, completely detached from the cosmic expansion, as H_0 drops out in $\sigma\Delta_1$. In this case, since Δ_1 is negative for look-back times, S_1/S_0 steadily decreases backwards in time, that is, with increasing look-back time $\tau_0 - \tau_1$. This need not be so if k varies. S_1/S_0 initially decreases in look-back time if ε is sufficiently small or negative, cf. after (2.18), which can readily be seen from (3.1) with the expansions (2.22) and (2.24) substituted. However, S_1/S_0 can reach a minimum within the relevant look-back interval of 4.7 Gyr, and subsequently increase in this interval. The condition for this to happen is a zero of $d(S_1/S_0)/d\tau_1$ in the range $-0.327 < \Delta_1 < 0$, more explicitly,

$$\frac{1}{H_0}\frac{\dot{k}_1}{k_0}(1 - \sigma K_1) + \frac{\sigma}{\lambda + 2}(k_1/k_0)^{\lambda+1} = 0. \tag{3.6}$$

Here, k_1/k_0 is defined in (3.2), and $\dot{k}_1/k_0 = \dot{H}(\tau_1)/H_0$, cf. (2.5) and (3.4). We use the rescaled look-back interval, $\Delta_1 = H_0 \cdot (\tau_1 - \tau_0)$, as variable in (3.6) instead of τ_1. As the zero of (3.6) sought for is small, we can use the ascending series expansions of k_1/k_0 and K_1 in (2.22) and (2.24), respectively. The series of $\dot{k}_1/k_0$

is most easily obtained by differentiating (2.22). In this way, we expand (3.6) as,

$$1 + A_1\Delta_1 + A_2\Delta_1^2 + A_3\Delta_1^3 + \cdots = 0, \tag{3.7}$$

$$A_1 := \frac{2(\lambda+2)c_2^{(1)} + \sigma\varepsilon}{\sigma - (\lambda+2)\varepsilon},$$

$$A_2 := \frac{-1}{\sigma - (\lambda+2)\varepsilon}\left[(\lambda+3)\sigma c_2^{(1)} + \frac{3(\lambda+2)c_3^{(1)}}{(H_0\tau_0)^3} + \frac{1}{2}\lambda\sigma\varepsilon^2\right],$$

$$A_3 := \frac{1}{\sigma - (\lambda+2)\varepsilon}\left[\frac{(2\lambda+5)\sigma c_3^{(1)}}{(H_0\tau_0)^3} + \frac{4(\lambda+2)c_4^{(1)}}{(H_0\tau_0)^4} + \frac{4}{3}\lambda(\lambda+2)\sigma c_2^{(1)}\varepsilon - \frac{1}{2}\lambda(\lambda+1)\sigma\delta\varepsilon + \mathrm{O}(\varepsilon^2)\right]. \tag{3.8}$$

The coefficients $c_i^{(1)}$ are the c_i in (2.23) taken at $\lambda = 1$, otherwise we put $\lambda \approx 7.5$, which is the scaling exponent of the luminosity, cf. (2.1). The solar parameter σ is defined in (2.21).

For the remainder of this section, we consider the expansion factor (3.3), discussed in Tables VIII–X. In (3.8), we insert $\lambda \approx 7.5$ as well as $H_0\tau_0$, ε, δ and the $c_i^{(1)}$ listed in Table VIII. The solar parameter σ is exemplified in Tables IX and X by two values $\sigma_{\mathrm{a,b}}$. Substituting $\sigma_{\mathrm{a}} \approx 0.774$ for σ in (3.8), we find $A_1 \approx 4.936$, $A_2 \approx -11.86$ and $A_3 \approx 17.28$. Solving (3.7), we obtain the look-back interval $\Delta_1(\sigma_{\mathrm{a}}) \approx -0.143$, so that the solar flux (3.1) attains its minimum at a look-back time of 2.06 Gyr. The convergence of (3.7) is quite good; dropping the third order term, we find a zero at -0.149. In the first row of Table X, we list the flux, luminosity and temperature ratios at 2.06 Gyr.

The second choice of solar parameter, $\sigma_{\mathrm{b}} \approx 1.22$, gives $A_1 \approx 3.169$, $A_2 \approx -8.395$ and $A_3 \approx 13.05$, so that the zero of (3.7) is $\Delta_1(\sigma_{\mathrm{b}}) \approx -0.191$, again the only one in the relevant range indicated before (3.6). The flux minimum then occurs at a look-back time of 2.75 Gyr, dealt with in the second row of Table X.

Table VIII. Parameters relating to the expansion factor $a(\tau) \propto \tau^{-1/2}\sinh((3/2)\tau/\tau_0)$, cf. (3.3). $\eta = 3/2$, $\alpha = 1$, and $\beta = -1/2$ are input, cf. (2.6). $q_0 = -1 + \varepsilon$ and $p_0 = 1 + \delta$ define the second and third order of the ascending series of $a(\tau)$, cf. (2.3). $H_0\tau_0$, ε, and δ are derived via (2.8)–(2.10). The inferred present epoch is $\tau_0 \approx 16.5$ Gyr, cf. after (2.5). The coefficients $c_i^{(\lambda=7.5)}$ determine the second to fourth order of the ascending series of K_1/Δ_1 in look-back time, cf. (2.24) and Table IX. The $c_i^{(\lambda=1)}$ are required in (3.8) for the solar flux minimum. These coefficients are calculated via (2.23), with the parameters in this table

$H_0\tau_0$	ε	δ	$c_2^{(\lambda=1)}$	$c_3^{(\lambda=1)}$	$c_4^{(\lambda=1)}$	$c_2^{(\lambda=7.5)}$	$c_3^{(\lambda=7.5)}$	$c_4^{(\lambda=7.5)}$
1.15719	-2.7862×10^{-3}	0.42449	0.20806	0.42403	0.45364	0.20809	0.41808	0.71616

Table IX. Input parameters from Table VIII, $\tau_0 - \tau_1$ is the look-back time. The first two look-back times are listed since the solar flux attains its minimum there, cf. Table X, subject to a time variation of the gravitational constant defined by the expansion factor in Table VIII, cf. (2.5) and (2.7). The solar flux minimum depends on the choice of the solar parameter σ in (3.1); it occurs at 2.06 Gyr if $\sigma \approx 0.774$ and at 2.75 Gyr for $\sigma \approx 1.22$, cf. after (2.21). The rescaled, dimensionless look-back interval $H_0 \cdot (\tau_0 - \tau_1)$ is denoted by $-\Delta_1$, as in Table VII. k_1/k_0 is the look-back/present-day ratio of the gravitational constant, and K_1 determines the luminosity ratio, cf. (3.1) and Table X. k_1/k_0 is calculated via (3.2), and K_1/Δ_1 via (2.24) with $\lambda = 7.5$, cf. (2.1), and the $c_i^{(\lambda=7.5)}$ in Table VIII

$\tau_0 - \tau_1$ (Gyr)	$-\Delta_1$	k_1/k_0	K_1/Δ_1
2.06	0.143	1.00478	1.01087
2.75	0.191	1.00937	1.02130
3.8	0.264	1.02038	1.04572
4.5	0.313	1.03120	1.06896

4. PLANETARY PALEOCLIMATES

Earth's present mean surface temperature T_0^S relates to its black-body equilibrium temperature as $T_0^S \approx T_0 + 33\,\text{K}$, where $T_0 \approx 255\,\text{K}$ and $T_0^S \approx 15°\text{C}$ (Sagan and Chyba, 1997; Kasting and Catling, 2003). We assume this shift of 33 K, due to the $H_2O - CO_2$ greenhouse effect, to hold in look-back time as well, $T_1^S \approx T_1 + 33\,\text{K}$. The look-back surface temperature can thus be recovered from the black-body ratio as $T_1^S(°\text{C}) \approx 255 \cdot T_1/T_0 - 240$. This temperature as well as the black-body ratios T_1/T_0 (always in K) are listed in Tables III, V–VII and X for various look-back times. The black-body temperature ratios are calculated via the flux and luminosity ratios (3.1), with (3.2) and (2.24) substituted.

The following temperature estimates, based on the expansion factor (3.3), are given for two values of the solar parameter σ in the luminosity ratio (3.1), $\sigma_a \approx 0.774$ and $\sigma_b \approx 1.22$, cf. after (2.21); a σ moderately outside this range would also qualify. At look-back times of 3.8 and 4.5 Gyr, we find the surface temperatures $T_1^S(3.8, \sigma_a) \approx 15°\text{C}$ and $T_1^S(4.5, \sigma_a) \approx 19°\text{C}$, cf. Table X, and σ_b generates $T_1^S(3.8, \sigma_b) \approx 8.8°\text{C}$ and $T_1^S(4.5, \sigma_b) \approx 12°\text{C}$. This is to be compared to the estimates for constant k, cf. after (3.5) and Table VII, where we find $T_1^S(3.8, \sigma_a) \approx 3.4°\text{C}$ and $T_1^S(4.5, \sigma_a) \approx 1.6°\text{C}$, whereas σ_b gives temperatures below the freezing point, $T_1^S(3.8, \sigma_b) \approx -2.2°\text{C}$ and $T_1^S(4.5, \sigma_b) \approx -4.8°\text{C}$.

Negative surface temperatures in the Archean and Hadean are rather unlikely. There is evidence for liquid water as early as 4.4 Gyr ago, inferred from oxygen isotope ratios in zircon grains, indicative of supracrustal material exposed to liquid water (Wilde *et al.*, 2001; Mojzsis *et al.*, 2001). There is likewise evidence for liquid water from sedimentary rocks deposited in the Early Archean, 3.6 − 3.9 Gyr ago (Nutman *et al.*, 1984, 1996). The early Archean Earth was presumably warmer than today, with average surface temperatures exceeding 15°C, as there are no traces of glaciation prior to 2.7 Gyr, most notably the absence of polar caps

Table X. Input parameters from Tables VIII and IX. $\tau_0 - \tau_1$ is the look-back time (input), L_1/L_0 the solar look-back/present-day luminosity ratio. S_1/S_0 and T_1/T_0 are the corresponding flux and black-body temperature ratios at the planetary orbit, cf. the discussion following (3.3). These ratios are calculated via (3.1) (with $\lambda = 7.5$) and Table IX, and listed for two choices of solar parameter, $\sigma_a \approx 0.774$ and $\sigma_b \approx 1.22$, cf. after (2.21). In the first case, the solar flux $S_1/S_0(\sigma_a)$ attains its minimum at 2.06 Gyr, generating a mean surface temperature of 11°C. This is the lowest temperature reached in look-back time with this solar parameter, only slightly lower than the present 15°C. The flux then increases in look-back time, generating a surface temperature of 19°C at 4.5 Gyr. The surface temperatures apply to Earth only, $T^S \approx T + 33$ K, where T is the black-body equilibrium temperature. Earth's greenhouse shift of 33 K is assumed constant in look-back time. If we use σ_b as solar parameter in the luminosity ratio (3.1), the minimum of $S_1/S_0(\sigma_b)$ occurs at 2.75 Gyr, from which a surface temperature of 7.2°C can be inferred. The temperature then increases during the Archean and Hadean, reaching 12°C at 4.5 Gyr. Earth's mean surface temperature stays well above the freezing point of water for a solar parameter in the range defined by $\sigma_{a,b}$, in contrast to the surface temperatures obtained with constant k, cf. Table VII

$\tau_0 - \tau_1$ (Gyr)	$L_1/L_0(\sigma_a)$	$S_1/S_0(\sigma_a)$	$T_1/T_0(\sigma_a)$	$T_1^S(\sigma_a)$ (°C)	$L_1/L_0(\sigma_b)$	$S_1/S_0(\sigma_b)$	$T_1/T_0(\sigma_b)$	$T_1^S(\sigma_b)$ (°C)
2.06	0.93210	0.9410	0.9849	11	0.88101	0.8894	0.9711	7.6
2.75	0.93174	0.9493	0.9871	12	0.86626	0.8826	0.9693	7.2
3.8	0.95855	0.9980	0.9995	15	0.87027	0.9061	0.9757	8.8
4.5	1.00015	1.0635	1.0155	19	0.89416	0.9508	0.9875	12

(Kasting, 1989). Further circumstantial evidence for a warm Archean Earth is the ^{34}S depletion in sediments dated back to $2.6 - 3.5$ Gyr, possibly caused by sulfur-reducing bacteria in $30 - 50°$C oceans (Ohmoto and Felder, 1987; Habicht *et al.*, 2002; Ono *et al.*, 2003). Similarly, ^{13}C-depleted carbon in 3.7 Gyr sedimentary rocks may be due to planktonic organisms (Mojzsis *et al.*, 1996; Rosing, 1999). Isotopic evidence for freshwater microorganisms $2.6 - 2.7$ Gyr ago is given in Watanabe *et al.* (2000). To explain the high surface temperatures throughout the Archean despite a dim Sun, an increased atmospheric CO_2 concentration supplemented by other greenhouse gases such as CH_4 and, in the Late Archean and Early Proterozoic, O_2 was invoked (Sagan and Chyba, 1997; Kasting and Catling, 2003; Tajika, 2003). Life may already have been present for most part of the Hadean, periodically extinguished whenever the oceans evaporated by asteroid impacts (Sleep *et al.*, 1989; Wilde *et al.*, 2001). If so, one can reckon that life is a common occurrence with a substantial effect on cosmic evolution (Dyson, 1979), which is one more reason to refrain from deterministic evolution equations in cosmology.

There is evidence for fluid water on Mars some 3.8 Gyr ago, at the end of the heavy-bombardment era, such as degrading impact craters and channel networks, valleys and canyons caved out by fluvial erosion (Carr, 1996). The possibility to reach surface temperatures above the melting point by CO_2 greenhouse warming was studied in Kasting (1991), where estimates for the required solar flux, depending on the CO_2 pressure, were derived. The weakest bound, attained at 5 bar, is $S_1/S_0 > 0.86$, where S_1 is the flux in look-back time, 3.8 Gyr, normalized with the present flux S_0 at the Martian orbit as in (3.1). Table VII indicates that this cannot be achieved with constant k. Deviations from a 5 bar surface pressure can even drive the required flux ratio beyond 0.9. The flux ratios in Table X, based on the expansion factor (3.3), still qualify, $S_1/S_0(\sigma_a) \approx 1.0$, and $S_1/S_0(\sigma_b) \approx 0.91$. The solar flux, subject to a varying k, even increases in the heavy-bombardment period, reaching $S_1/S_0(\sigma_a) \approx 1.06$ and $S_1/S_0(\sigma_b) \approx 0.95$ at 4.5 Gyr. CO_2 ice clouds can increase the greenhouse effect by backscattering of thermal IR radiation (Forget and Pierrehumbert, 1997; Mischna *et al.*, 2000). In this way, temperatures above the freezing point can be reached, despite the higher albedo these clouds would generate. A greenhouse warming of early Mars is further complicated by a CO_2 recycling problem due to the absence of volcanism, though other greenhouse gases like methane could compensate for that.

The expansion factor (3.3) is an attractive choice for the reasons summarized below, but not the only possible one. In Tables II–VI, we study the parameter space (η, α, β) defining the expansion factors (2.6). We restrict these parameters by requiring surface temperatures and flux ratios capable of solving the faint young Sun paradox without modifications of the present $H_2O - CO_2$ greenhouse effect, the 33 K shift, that is. Further bounds can be obtained from the redshift scaling of galaxy diameters discussed below. Moreover, there is no necessity to restrict to the analytic shape (2.6) of the expansion factor, we could have proceeded with the ascending series (2.3) and (2.11). We chose the expansion factors (2.6) to have an analytically tractable cross-over between power-law and exponential expansion, where the parameters (η, α, β) prescribe the asymptotics of the early and final stage, cf. after (2.6).

In Tables III, V and VI, we study the impact of the cosmic age on the solar luminosity evolution. We consider a look-back time of 3.8 Gyr, the borderline of Archean and Hadean era, and we take $\varepsilon = 0$ as input, so that the present-day logarithmic derivative $\dot{k}_0/k_0$ vanishes, cf. (2.18). The current bounds on $\dot{k}_0/k_0$ are such that this derivative cannot affect the solar luminosity evolution in a noticeable way, cf. the discussions following (2.18) and (3.5). In Table III, η is arbitrarily prescribed, ranging between 0.5 and 3.5. α and β are calculated from (2.13) and (2.14), with input parameters $\varepsilon = 0$ and $H_0\tau_0 = 1.1$, the latter means a present epoch of $\tau_0 \approx 15.8$ Gyr, cf. after (2.5). δ is calculated from (2.12). If we take $\sigma_a \approx 0.774$ as solar parameter in the luminosity ratio (3.1), the surface temperatures range from 24°C for $\eta = 0.5$ to 5.7°C for $\eta = 3.5$, as inferred from the black-body temperature ratios explained at the beginning of this section. The respective fluxes at the planetary orbits are 15% higher (14% lower for $\eta = 3.5$) than the present-day flux. (The quoted surface temperatures refer to Earth, whereas the flux and temperature ratios apply to any planet.) Using $\sigma_b \approx 1.22$ instead, we find surface temperatures between 17 °C and 0°C, and fluxes 4% higher (22% lower) than presently. Fluxes some 20% lower than the present one are not an attractive option, as they cannot significantly raise the surface temperature above the freezing point of water, so that an enhanced greenhouse effect and/or a lower albedo would still be necessary for a warm paleoclimate. Thus we can restrict the η-range to $\eta \leq 2.5$ if we choose σ_b as solar parameter in (3.1).

Table V is likewise compiled at a look-back time of 3.8 Gyr, but for a lower cosmic age, $H_0\tau_0 = 0.95$ or $\tau_0 \approx 13.7$ Gyr, otherwise the input parameters of Table III are retained. If we reduce the cosmic age, the effect of the time variation of k gets more pronounced, as the look-back time moves into the cross-over regime of the expansion factor (2.6). At $\tau_0 \approx 13.7$ Gyr, and for a look-back-time of 3.8 Gyr, the third and fourth order terms of the expansion (2.24) already give a noticeable contribution. The surface temperatures based on σ_a range between 32°C ($\eta = 0.5$) and 6.9°C ($\eta = 3.5$), they are generated by fluxes 30% higher (12% lower for $\eta = 3.5$) than the present-day flux. If the luminosity ratio is calculated with σ_b, we obtain surface temperatures between 26°C and 1.1°C, and the respective solar fluxes are 18% higher (20% lower) than the present flux. A 20% reduction gives rather low surface temperatures, and fluxes exceeding the present flux by more than 10% can lead to a run-away atmosphere (Kasting and Catling, 2003). Thus, in case of σ_a, a safe η-range is 2 to 3.5, with temperatures in the range 17°C to 6.9°C, and fluxes 4% higher (12% lower) than the present flux. In case of σ_b, we obtain a similar temperature and flux range for η between 1.5 and

2.5. If the cosmic age is further reduced, temperature and flux quickly increase, and the admissible η-interval shrinks to maintain a stable greenhouse effect. However, a cosmic age of $\tau_0 \approx 13.7$ Gyr is already rather low, if one considers age estimates from nuclear chronometers (which are unaffected by a time variation of k). The averaged Th/Eu age of three M15 giants is estimated in Sneden *et al.* (2000) as 14 ± 3 Gyr, and the age of the halo star CS 31082–001 is quoted in Schatz *et al.* (2002) as 15.5 ± 3.2 Gyr, inferred from U/Th ratios. Such age estimates have constanly been revised downwards in recent years (Truran *et al.*, 2002), to bring them in line with the current cosmological standard model (Tegmark *et al.*, 2004).

In Table VI, we consider $H_0\tau_0 = 1.3$ (or $\tau_0 \approx 18.7$ Gyr, four times the solar age), but otherwise the same parameters as in Tables III and V. A high cosmic age diminishes the contribution of the nonlinear terms in the expansions (2.22) and (2.24), so that the variation of k has a lesser impact on the luminosity evolution in geological look-back time, which shows in low surface temperatures. The temperatures obtained with σ_a range between 17°C ($\eta = 0.5$) and 4.8°C ($\eta = 3.5$), and the respective fluxes at the planetary orbits are 3% higher (15% lower) than presently. As for σ_b, we find surface temperatures between 11°C and −0.8°C, and solar fluxes 6% lower (23% lower) than the present flux. Thus, if we use σ_b as solar parameter, we have to restrict the η-range to $\eta < 2$ to obtain surface temperatures above 4°C at 3.8 Gyr.

In Tables III, V and VI, we also list the scaling exponent of the angular diameter of radio galaxies at high z, $\theta \propto z^{-(n-1)}$, cf. after (2.17). This exponent, $n - 1$, increases with η and decreases with $H_0\tau_0$ as long as it stays positive. A negative $n - 1$ means constant angular diameters, $\theta \propto 1$, at high redshift, cf. the discussion after (2.17). The latter is not really ad odds with observations, but increasingly unlikely, cf. the n-values cited before (2.18), which tend toward $n \approx 2$. It is evident from these tables that an n close to 2 severely limits the possible η-range.

The expansion factor (3.3) is remarkable as it gives a very small $\varepsilon \approx -2.786 \times 10^{-3}$, cf. (2.9) and (2.18), without any need for a finetuning of α, β and η. To contrast this, I mention two other expansion factors, which come to mind when looking at Tables III and VI. The first is defined by $\eta = 2$, $\alpha = 3/4$ and $\beta = -1/4$ in (2.6), resulting in $H_0\tau_0 \approx 1.306$ and $\varepsilon \approx -0.013$, cf. (2.8) and (2.9). At a look-back time of 3.8 Gyr, surface temperatures between 4°C and 10°C (depending on the solar parameter σ) can be read off from Table VI. The second expansion factor is specified by $\eta = 1$, $\alpha = 2$, $\beta = -3/2$, so that $H_0\tau_0 \approx 1.126$ and $\varepsilon \approx -0.041$, with surface temperatures in the range 14–21°C, cf. Table III. In the first case, ε comes close to the helioseismological bound, cf. after (2.18). In the second example, this bound is even exceeded, though a slight variation of η can still make ε arbitrarily small.

The expansion factor (3.3) is capable of producing a surface temperature 3.8 Gyr ago which is way above the freezing point. Our estimates range between 9°C and 15°C, cf. Table X, depending on the choice of the solar parameter

σ, and the temperature even moderately increases during the Hadean, reaching 12–19°C at 4.5 Gyr. The corresponding temperature range for constant k is -2 to 3°C at 3.8 Gyr, and -5 to 2°C at 4.5 Gyr, cf. Table VII, subject to an invariable 33 K greenhouse effect. In case of a varying k, the lowest surface temperatures, 7–12°C, are attained in the Early Proterozoic and Late Archean, 2–2.8 Gyr ago, cf. Table X. This compares to 2–8°C for constant k in the same epoch, cf. Table VII. The surface temperatures generated by the expansion factor (3.3) are quite in line with the mentioned evidence for a warm paleoclimate, in particular with the existence of warm oceans in the Archean and Hadean, and with the existence of fluid water on Mars before 3.8 Gyr ago. Finally, the age estimates of the halo giants cited above can convincingly be accommodated in a 16.5 Gyr universe.

5. CONCLUSION

The luminosity of the early Sun can be increased by a cosmic time variation of the gravitational constant. The basic concepts to argue this are Schwarzschild's scaling relation (2.1) and the constancy of the moderate dimensionless ratio (2.4) in the cosmic evolution. The quantitative modeling of the luminosity evolution subject to a varying gravitational constant depends on very few input parameters. We do not need to invoke climatic changes, which hinge upon many details such as the actual greenhouse gases involved or the composition of the albedos. In case of Mars, for instance, it is uncertain if there was a significant albedo of CO_2 ice clouds over a significant period before 3.8 Gyr, because of the lack of volcanic CO_2 recycling (Kasting and Catling, 2003). Even if this is taken for granted, it may have resulted in a warming or cooling of the Martian surface, depending on the actual size of the ice particles, which determines whether the backscattering of the outgoing infrared outweighs the reflection of the incident solar wavelengths (Forget and Pierrehumbert, 1997).

The time variation of k can severely impact planetary paleoclimates, even though the present variation is very small as suggested by lunar laser ranging and helioseismological bounds. To put this beyond doubt, we assumed the logarithmic time derivative of k to be negligible at the present epoch. The crucial point is to take the higher order derivatives into account on geological time scales. The solar age is about one third to one fourth of the cosmic age, and on that scale higher orders in the ascending series of k do matter in the stellar luminosity evolution, since the cross-over regime of the cosmic expansion factor is reached. Quantitative estimates with regard to the effect of the cosmic age on the solar flux and in particular on Earth's surface temperature have been given in the tables and in Section 4. Typical mean surface temperatures in the Archean and Hadean range within 7–20°C.

Finally, a time variation of k also bears on the redshift scaling of galaxy diameters. We can relate the scaling exponent of radio galaxies in the high-z

regime to the power-law asymptotics of the cosmic expansion factor. Based on this scaling index, we study the solar flux and Earth's paleoclimate at critical look-back times in Tables VIII–X. We find a qualitative change in the solar flux evolution, effected by the time variation of k. If k is kept constant, the solar flux decreases in look-back time. In case of a varying gravitational constant, however, the solar flux reaches a minimum and then increases during most part of the Archean and throughout the Hadean period, which results in surface temperatures prior to 3.8 Gyr ago that are quite comparable to the present 15°C, cf. Table X. This gives further credence to the existence of oceans in the Hadean, and suggests that the solar flux at the Martian orbit was capable of sustaining a CO_2 greenhouse effect strong enough for the large-scale presence of liquid water.

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